Supplemental Material: Position-dependent Importance Sampling of Light Field Luminaires

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Abstract—This complementary document provides the reader with more details on the derivation of the different equations contained in the paper entitled "Position-dependent Importance Sampling of Light Field Luminaires".

Index Terms—Models of Light Sources, Light Field, Importance Sampling, Real-time Rendering,

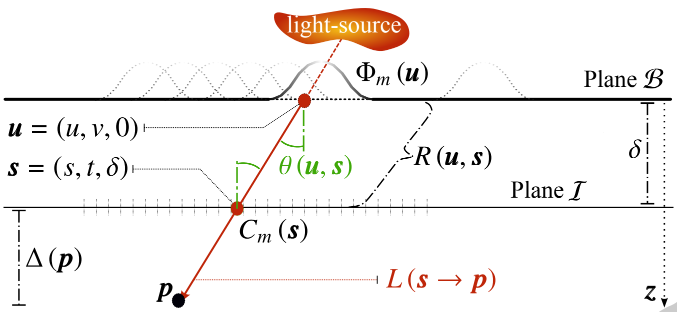

Fig. 1. Model parametrization of [1]. The 4D space of rays emitted from the light source is parametrized by a position $\mathbf{u}$ on a plane $\mathcal{U}$ that supports the reconstruction 2D basis functions Φ_m, and a position $\mathbf{s}$ on a plane $\mathcal{S}$, that supports the C_m images. $\Phi_m(\mathbf{u})$ is a short notation for $\Phi_m(u,v)$ and $C_m(\mathbf{s})$ for $C_m(s,t)$. δ is the inter-plane distance and $\Delta(\mathbf{p})$ is the distance between $\mathbf{p}$ and $\mathcal{S}$. $\mathbf{u}$, $\mathbf{s}$ and $\mathbf{p}$ are aligned.

1 From Goesele's Model to Equation 1

In their paper, Goesele et al. describe their (cf. Equation 8 in [1]) light field luminaire by the following equation (using the notation introduced in Figure 1):

$$L(\mathbf{u} \to \mathbf{s}) = \frac{R^2(\mathbf{u},\mathbf{s})}{\cos^2\theta(\mathbf{u},\mathbf{s})} \sum_m C_m(\mathbf{s})\,\Phi_m(\mathbf{u})$$

with $L(\mathbf{u} \to \mathbf{s})$ representing the radiance transfered from $\mathbf{u}$ to $\mathbf{s}$. To simplify the notation, reader can note that the geometric configuration leads to

$$R(\mathbf{u},\mathbf{s}) = |\mathbf{u} - \mathbf{s}|$$

and to

$$\delta = |\mathbf{u} - \mathbf{s}| \cos\theta(\mathbf{u},\mathbf{s}) \, .$$

By combining these three equations together we obtain the equation used in our paper:

$$L(\mathbf{u} \to \mathbf{s}) = \frac{|\mathbf{s} - \mathbf{u}|^4}{\delta^2} \sum_m C_m(\mathbf{s})\,\Phi_m(\mathbf{u}) \, . \tag{1}$$

2 Derivation for $I_m(\mathbf{p})$

The irradiance $I(\mathbf{p})$ that can potentially reach $\mathbf{p}$ from the light source is:

$$I(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} L(\mathbf{s} \to \mathbf{p}) \frac{\cos\theta(\mathbf{u},\mathbf{s})}{|\mathbf{s} - \mathbf{p}|^2} \, d\mathbf{s}$$

According to the geometric configuration introduced in Figure 1, $\Delta(\mathbf{p}) = |\mathbf{s} - \mathbf{p}| \cos\theta(\mathbf{u},\mathbf{s})$. Therefore, it can also be written as presented in our paper:

$$I(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} L(\mathbf{s} \to \mathbf{p}) \frac{\Delta(\mathbf{p})}{|\mathbf{s} - \mathbf{p}|^3} \, d\mathbf{s} \tag{2}$$

Combining Equation 1 with Equation 2 leads to

$$I(\mathbf{p}) = \sum_m I_m(\mathbf{p})$$

$$I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} \frac{|\mathbf{s} - \mathbf{u}|^4}{\delta^2} \frac{\Delta(\mathbf{p})}{|\mathbf{s} - \mathbf{p}|^3} C_m(\mathbf{s})\,\Phi_m(\mathbf{u}) \, d\mathbf{s} \, .$$

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We replace $\Delta(\mathbf{p})$ by $|\mathbf{s}-\mathbf{p}|\cos\theta(\mathbf{u},\mathbf{s})$ and δ by $|\mathbf{u}-\mathbf{s}|\cos\theta(\mathbf{u},\mathbf{s})$ to obtain

$$\begin{aligned}
& I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} \frac{|\mathbf{s}-\mathbf{u}|^4\,|\mathbf{s}-\mathbf{p}|\cos\theta(\mathbf{u},\mathbf{s})}{|\mathbf{u}-\mathbf{s}|^2\cos^2\theta(\mathbf{u},\mathbf{s})\,|\mathbf{s}-\mathbf{p}|^3}\, C_m(\mathbf{s})\,\Phi_m(\mathbf{u})\,d\mathbf{s} \\
\Leftrightarrow\ & I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} \frac{|\mathbf{s}-\mathbf{u}|^2\,|\mathbf{s}-\mathbf{p}|\cos\theta(\mathbf{u},\mathbf{s})}{\cos^2\theta(\mathbf{u},\mathbf{s})\,|\mathbf{s}-\mathbf{p}|^3}\, C_m(\mathbf{s})\,\Phi_m(\mathbf{u})\,d\mathbf{s} \\
\Leftrightarrow\ & I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} \frac{|\mathbf{s}-\mathbf{u}|^2\,|\mathbf{s}-\mathbf{p}|\cos^2\theta(\mathbf{u},\mathbf{s})}{\cos^3\theta(\mathbf{u},\mathbf{s})\,|\mathbf{s}-\mathbf{p}|^3}\, C_m(\mathbf{s})\,\Phi_m(\mathbf{u})\,d\mathbf{s} \\
\Leftrightarrow\ & I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} |\mathbf{s}-\mathbf{p}|\,\frac{|\mathbf{s}-\mathbf{u}|^2\cos^2\theta(\mathbf{u},\mathbf{s})}{\cos^3\theta(\mathbf{u},\mathbf{s})\,|\mathbf{s}-\mathbf{p}|^3}\, C_m(\mathbf{s})\,\Phi_m(\mathbf{u})\,d\mathbf{s} \\
\Leftrightarrow\ & I_m(\mathbf{p}) = \int_{\mathbf{s}\in\mathcal{I}} |\mathbf{s}-\mathbf{p}|\,\frac{\delta^2}{\Delta^3(\mathbf{p})}\, C_m(\mathbf{s})\,\Phi_m(\mathbf{u})\,d\mathbf{s}
\end{aligned}$$

Since $\Delta(\mathbf{p})$ and δ do not depend on $\mathbf{s}$, we finally obtain the equations introduced in our paper:

$$I(\mathbf{p}) = \sum_m I_m(\mathbf{p}) \tag{3}$$

$$I_m(\mathbf{p}) = \frac{\delta^2}{\Delta^3(\mathbf{p})} \int_{\mathbf{s}\in\mathcal{I}} |\mathbf{s}-\mathbf{p}|\; C_m(\mathbf{s})\;\Phi_m(\mathbf{u})\;d\mathbf{s}\,. \tag{4}$$

References

[1] M. Goesele, X. Granier, W. Heidrich, and H.-P. Seidel, “Accurate light source acquisition and rendering,” *ACM Trans. Graph.*, vol. 22, no. 3, pp. 621–630, 2003.